PHYSICAL REVIEW E **81**, 017203 (2010)

# Quantum chaos and operator fidelity metric

Paolo Giorda[1] and Paolo Zanardi[1,2]
[1]*Institute for Scientific Interchange, Viale Settimio Severo 65, I-10133 Torino, Italy*
[2]*Department of Physics and Astronomy, Center for Quantum Information Science and Technology, University of Southern California, Los Angeles, California 90089-0484, USA*
(Received 6 March 2009; published 26 January 2010)

We show that the recently introduced operator fidelity metric provides a natural tool to investigate the crossover to quantum chaotic behavior. This metric is an information-theoretic measure of the global stability of a unitary evolution against perturbations. We use random matrix theory arguments to conjecture that the operator fidelity metric can be used to discriminate phases with regular behavior from quantum chaotic ones. A numerical study of the onset of chaotic in the Dicke model is given in order to support the conjecture.

DOI: 10.1103/PhysRevE.81.017203 PACS number(s): 05.45.Mt, 03.67.−a, 64.70.Tg

## I. INTRODUCTION

The information-theoretic task of distinguishing quantum states can be formulated in terms of the differential-geometric notion of *metric* over the quantum state space both in the pure and mixed state case [1]. This remarkable fact lies at the root of the recently proposed *fidelity approach* to quantum phase transitions (QPTs) [2]. The intuition behind this approach is quite simple: at the transition points a slight change in the system driving parameters, e.g., external fields, gives rise to a major structural change in the associated (thermal or ground) quantum state and this, in turn, should result in an enhanced statistical distinguishability. In view of the above mentioned connection one expects detectable signature in the metric function, e.g., singularities in the thermodynamical limit both at zero and finite temperature [3,4].

In Ref. [5] the ground-state approach has been extended to the operator level. In order to do that one has simply to notice that finite-time quantum evolutions over the Hilbert space $\mathcal{H}$ correspond to unitary operators that belong to a space, i.e., the space of linear operators $\mathcal{L}(\mathcal{H})$, that can be turned itself into a Hilbert one in many different ways. This allows one to introduce the notion of *operator fidelity* and the associated metric [5].

The main goal of this paper is to attempt to use operator fidelity to investigate another fascinating and somewhat elusive phenomenon: quantum chaos [7]. In many years of theoretical efforts a variety of approaches, such as random matrix theory (RMT) [8], quantum motion reversibility [9], stability [10], state fidelity [12], and entanglement [13] up to very recent measures of phase-space growth rate have been proposed [11]. All of these approaches appear to be successful in addressing specific aspects of the quantum chaotic behavior.

In the following we carry out our analysis along two well separated though mutually supporting paths. In the first analytical part we explore the consequences of the physical intuition that is behind the approach that we advocate: the quantum chaotic evolutions can be characterized as those who have resilience against random perturbations [12]. This resilience is here quantified by the information-theoretic distance between perturbed and unperturbed evolutions. We will make use of random matrix theory [8] arguments to formulate and support our main result on operator metric (in)sensitivity of (chaotic) regular system to random perturbations. The second part of our paper offers an implementation these ideas in a particular system. More precisely, we numerically analyze the crossover to quantum chaos in the Dicke model [14,15] and we show how the operator metric approach can be used as a tool to identify and characterize this crossover. We will conclude by a summary and an outlook.

## II. INFORMATION-THEORETIC METRIC OVER MANIFOLDS OF UNITARIES

Let us start by providing the setup and the basic facts about operator fidelity metric found in [5]. Let $\mathcal{H}$ be a quantum state space and $\rho$ a density matrix over it ($\rho \geq 0$, $\mathrm{tr}\,\rho = 1$). One can define the following Hermitian product over $\mathcal{L}(\mathcal{H})$: $\langle X, Y\rangle_\rho := \mathrm{tr}(\rho X^\dagger Y)$. If $\rho > 0$ then this scalar product is nondegenerate and $\|X\|_\rho := \sqrt{\langle X, X\rangle_\rho}$ defines a norm over $\mathcal{L}(\mathcal{H})$. The *operator fidelity* $X, Y \in \mathcal{L}(\mathcal{H})$ is then given by

$$\mathcal{F}_\rho(X,Y) := |\langle X, Y\rangle_\rho|. \tag{1}$$

This quantity has well-defined operational meaning in terms of statistical distinguishability (the higher $\mathcal{F}$ the lower this latter) in terms of bipartite quantum states [6], interferometric schemes associated with $X, Y$ and $\rho$ and decoherence rates [5]. The case we will be concerned with is when $X = U_\lambda := e^{-itH_\lambda}$, $Y = U_{\lambda'} (\lambda \neq \lambda')$ we will also assume that $[\rho, H_\lambda] = 0$.

In passing we would like to note that Eq. (1) has relevance to nonequilibrium dynamics, e.g., quenches where one goes suddenly from $H_\lambda$ to $H'_\lambda$ [19]. Moreover, Eq. (1) contains as a special case the Loschmidt echo $|\langle 0|e^{-itH_{\lambda'}}|0\rangle|$, pointing out the relevance of the operator fidelity to dephasing experiments [5,16]. In this Brief Report, following the differential-geometric spirit of [2] we are going to consider the operator fidelity [Eq. (1)] between infinitesimally different unitaries, i.e., $\lambda' = \lambda + d\lambda$. The proof of the following proposition is just a direct calculation analogous to the one performed at the state-space level [2]. Let $\{U_\lambda\} \subset \mathcal{U}(\mathcal{H})$ be a smooth family of unitaries over $\mathcal{H}$ parametrized by elements $\lambda$ of a manifold $\mathcal{M}$. One finds $\mathcal{F}_\rho(U_\lambda, U_{\lambda+\delta\lambda}) = 1 - \delta\lambda^2/2\chi_\rho(\lambda)(U', U')$, where $U' = \partial U/\partial\lambda$ and

$$\chi_\rho(\lambda)(X,Y) := \langle X, Y\rangle_\rho - \langle X, U_\lambda\rangle_\rho \langle U_\lambda, Y\rangle_\rho. \tag{2}$$

We see that the leading term in the expansion of Eq. (1) defines a quadratic form over the tangent space of the projective $\mathcal{L}(\mathcal{H})$ at $U_\lambda$. For full rank $\rho$ that quadratic form is a *metric*. This operator metric (also referred to as operator fidelity susceptibility in [5]) is the central tool of this Brief Report. The intuitive meaning of $\chi$ is quite simple: the larger the operator metric at $U_\lambda$ the greater the degree of statistical distinguishability between the quantum evolutions associated with two slightly different set of control parameters $\lambda$ and $\lambda+\delta\lambda$. This fact has been used in [5] to use Eq. (2) to study quantum criticality.

For the developments of this paper it is convenient to make use of the *superoperator* formalism common in quantum information science. We define over $\mathcal{L}(\mathcal{H})$ the superoperator $L_H:=[H,\bullet]$. This is nothing but the generator of the Heisenberg evolution and it is easy to see that, if $|n\rangle$s and the $E_n$s denote the eigenvectors and eigenvalues of $H(\lambda)$, $E_n-E_m$ and $|n\rangle\langle m|$ are the eigenvalues and eigenoperators of $L_H$, respectively. It follows that $|\hat{\Psi}_{n,m}\rangle=\rho_{m,m}^{-1/2}|n\rangle\langle m|$ it is an *orthonormal* basis of $\mathcal{L}(\mathcal{H})$, i.e., $\langle\hat{\Psi}_{n,m}|\hat{\Psi}_{p,q}\rangle=\delta_{n,p}\delta_{m,q}$. If we define $P=\sum_n|\hat{\Psi}_{n,n}\rangle\langle\hat{\Psi}_{n,n}|$ as the projection over the kernel of $L_H$, $Q=\mathbb{1}-P$, and $\delta_t(x):=[\sin(xt/2)/(x/2)]^2$, one finds that the operator metric is given by the sum of two distinct terns: $\chi_\rho=\chi_\rho^{(1)}+\chi_\rho^{(2)}$, where

$$\chi_\rho^{(1)}=\|\delta_t(L_H)Q|H'\rangle\|_\rho^2;\quad \frac{\chi^{(2)}}{t^2}=\|P|H'\rangle\|_\rho^2-|\langle\mathbb{1},H'\rangle_\rho|^2. \quad (3)$$

### III. OPERATOR FIDELITY AND RANDOM MATRIX THEORY

From the last equations we see that, in general $\chi_\rho$ depends on $t$, $H_\lambda$ and on its derivative $H'=\partial H_\lambda/\partial\lambda$. We would like to start our analysis of the chaos-related properties of $\chi_\rho$ by introducing a $t$-dependent quantity $\hat{\chi}_\rho$ that *contains information just on the Hamiltonian* $H_\lambda$. A possible way to achieve this goal is (i) replace $H'$ in the operator metric by a perturbation $V$ that is assumed to be drawn by a Gaussian ensemble, with measure $\mathcal{D}[V]$, of *random matrices* [8], (ii) take $\hat{\chi}_\rho(t,H):=\int\mathcal{D}[V]\chi_\rho(t,H,V)$ as the average $\chi_\rho$ over the ensemble of $V$s.

More precisely, we will assume that the perturbation matrix elements $V_{n,m}$ are Gaussian random variables satisfying the relations $\int\mathcal{D}[V]V_{n,m}\bar{V}_{p,q}\sim\delta_{n,p}\delta_{m,q}$. The key step is the following fact: if $R_\rho$ denotes the superoperator $X\to X\rho$ one has $\int\mathcal{D}[V]|V\rangle\langle V|\sim R_\rho$ [20]. Now, by using this result and writing $\hat{\chi}_\rho^{(1)}=\mathrm{Tr}[|V\rangle\langle V|\delta_t(L_H)Q]$ and $\hat{\chi}_\rho^{(2)}=\mathrm{Tr}[|V\rangle\langle V|(P-|\mathbb{1}\rangle\langle\mathbb{1}|)]$, one finds

$$\hat{\chi}_\rho^{(1)}(t,H)\sim\mathrm{Tr}[R_\rho\delta_t(L_H)Q],\quad \hat{\chi}_\rho^{(2)}(t,H)\sim t^2(1-\mathrm{tr}\,\rho^2).$$

From these relations we clearly see that the two terms of the operator fidelity metric behave quite differently upon averaging over the perturbation $V$: on the one hand $\hat{\chi}_\rho^{(2)}$ looses any direct connection with the Hamiltonian $H$, just the purity of the state $\rho$ is returned, on the other hand the averaged $\hat{\chi}_\rho^{(1)}$ still shows an highly nontrivial dependence on the spectral features of $H$. This explicit dependence is the major feature of our operator fidelity analysis that differs from the previous state fidelity approaches of Ref. [12]. This feature allows for a new approach to the study of the transition to chaos. In the rest of the Brief Report we are going to focus on this latter term $\hat{\chi}_\rho^{(1)}$.

Let us make the content of the $\hat{\chi}_\rho^{(1)}$ explicit by resorting once again to the eigenoperator basis $|\hat{\Psi}_{n,n}\rangle$ of $L_H$, from Eq. (3) we find $\hat{\chi}_\rho^{(1)}(t,H)\sim\sum_{n\neq m}\rho_{n,n}\delta_t(\Delta_{n,m})$ $\Delta_{n,m}:=E_n-E_m$. This equation shows that the perturbation averaged $\chi^{(1)}$ depends on the of level spacing distribution of $H$. In particular—in view of the property $\lim_{t\to\infty}t^{-1}\delta_t(x)=2\pi\delta(x)$—for sufficiently large $t$ we expect contributions from small $\Delta_{n,m}$, i.e., by almost crossing levels, to dominate the operator metric. The key observation at this point is that one of the possible characterization of the presence of chaos in quantum system stems from the *level spacing analysis* [7,15]. In this context, the properly normalized spectrum [17] is analyzed in terms of the spacing between consecutive energy levels $S_n=E_{n+1}-E_n$. If $P(S)$ denotes the probability that two nearest-neighboring levels have an energy difference $S$ then the behavior $P(S)$ for $S\to 0$ encodes the main features distinguishing a chaotic system from a regular one. Indeed, the distributions characterizing the regular spectra are Poissonian, allowing for a nonvanishing probability to have $S=0$ spacing, i.e., level crossings. In the chaotic case one has Wigner-like distributions $P_W(S)\propto S^\nu\exp(-S^2)$; their small spacing behavior, in view of symmetry, is dominated by $S^\nu$, with $\nu>0$. This entails for the phenomenon of *level repulsion*, i.e., suppressed level crossings.

The above remarks together put us now in the position to formulate the main result of this Brief Report in the form of a conjecture: *for sufficiently large t the operator metric* $\chi_\rho^{(1)}$ *is finite for regular quantum Hamiltonian and (almost) vanishing for quantum chaotic ones*. To provide further support to this conjecture let us consider the *average* behavior of $\hat{\chi}_\rho^{(1)}$ with respect $H$, i.e., $\tilde{\chi}_\rho^{(1)}(t):=\int\mathcal{D}[H]\hat{\chi}_\rho^{(1)}(t,H)$. Since $\hat{\chi}_\rho^{(1)}(t,H)$ is a spectral function integration over the eigenvectors of $H$ can be readily performed and one is left with [8]

$$\tilde{\chi}_\rho^{(1)}(t)\sim\int e^{-\sum_n E_n^2}\prod_{n<m}dE_n dE_m(E_n-E_m)^\nu\hat{\chi}_\rho^{(1)}(t,H).$$

The exponent $\nu$ tells apart ensembles of regular, $\nu=0$, from chaotic, $\nu>0$ Hamiltonians. From the last equation and by resorting to the explicit representation $\hat{\chi}_\rho^{(1)}$ above, we conclude $\lim_{t\to\infty}\tilde{\chi}_\rho^{(1)}(t)/t\neq 0$ for regular $H$ and $\lim_{t\to\infty}\tilde{\chi}_\rho^{(1)}(t)/t\approx 0$ for chaotic $H$. Assuming *typicality*, i.e., typical and average behavior coincide, in $H$ these Eqs are nothing but a formulation of the *conjecture* above.

Of course none of the above arguments is rigorous, for as they depend on a set of unproven assumptions, e.g., RMT ensembles physical relevance, typicality, moreover they somehow neglect the potential sensitivity on $\rho$ and finally rely on a "naive" large $t$ limit. Nevertheless the conceptual content of the *conjecture* is quite compelling and intuitive at the same time: typical regular Hamiltonians have a much higher susceptibility against random perturbations than chaotic ones. The analysis confirms the results on the chaotic behavior carried on with different approaches [12] and allows to root them in the general theoretical framework supplied by operator fidelity.

## IV. OPERATOR FIDELITY AND TRANSITION TO CHAOS: THE DICKE MODEL

In this section we complement the above theoretical analysis and use the fidelity metric as a tool to study the transition from regular to a chaotic regime in the Dicke model [14,15]. This model consists of a set of $N$ identical spin 1/2 atoms with atomic level splitting $\omega_0$ placed in an ideal cavity that collectively interact with a single bosonic field described by the operators $\{a,a^\dagger\}=1$ and characterized by the frequency $\omega=\omega_0$ (resonance). The Dicke Hamiltonian reads

$$H=\omega_0 J_z+\omega a^\dagger a+\frac{\lambda}{\sqrt{2j}}(a^\dagger+a)(J_+ + J_-), \quad (4)$$

where $J_z=\Sigma_{i=1}^N \sigma_i^z$ and $J_\pm=J_x\pm J_y=\Sigma_{i=1}^N \sigma_i^\pm$ are collective spin operators. The pseudospin length is fixed ($j=N/2$), and we have that the interaction is weighted by $1/\sqrt{2j}=1/\sqrt{N}$. The Dicke model is integrable at finite sizes only if, for small $\lambda$, one implements the rotating wave approximation (RWA), i.e., if one neglects the terms $a^\dagger J_+, aJ_-$. In the thermodynamical limit (TDL) $N\to\infty$ [14,15] it is integrable and it exhibits a quantum phase transition in correspondence of $\lambda=0.5$. This critical value separates the "normal" phase ($\lambda<0.5$) and the "super-radiant" phase and the related phase transition can be appropriately described in the context of the state fidelity approach [2].

Here we are interested to the finite size instance of the model that, in absence of the RWA is not integrable and exhibits a regular to chaos crossover. Indeed, the only (known) symmetry property is described by the parity operator $\mathbf{P}=\exp[\mathbf{i}\pi\hat{\mathbf{N}}]$, where $\hat{N}=a^\dagger a+J_z+j$ is the operator that counts the number of total quanta present in the system. In order to characterize the quantum chaotic behavior of Eq. (4) one can then resort to the study of the properties of the spectrum relative to the odd (even) subspaces. Indeed, in [15] the authors showed that in correspondence of the value of $\lambda=0.5$, which is critical in the TDL, one can observe a transition from a regular region ($\lambda<0.5$) characterized by Poisson-like level spacing distributions to a chaotic region ($\lambda>0.5$) where the distributions are Wigner-like with $\nu=1$.

The actual system we used in our simulations is a set of $N=20$ atoms coupled with a bosonic bath. To make computations feasible we have to introduce a cutoff in the—infinite dimensional—bosonic state space. This cutoff has to be chosen in such a way that the bosonic system still operates as a bath for the atoms. We thus choose to include the first $M=128$ bosonic energy levels; the total Hilbert space have thus dimension $d=4032$.

Let us first consider the level spacing distribution $P_\lambda(S)$ of $H(\lambda)$. A characterization of $P_\lambda(S)$ can be given by its statistical distinguishability with respect to the Wigner distribution $P_W(S)(\nu=1)$; this statistical distance can be measured by the relative entropy $\mathcal{S}(P_\lambda \| P_W) \coloneqq \Sigma_n P_\lambda(S_n)\log[P_\lambda(S_n)/P_W(S_n)]$ [18]. In the inset of Fig. 1(a) we see that in the chaotic region the relative entropy is uniform and small, i.e., $P_\lambda(S) \approx P_W(S)$, whereas in the regular one we have in general higher values and a more complex behavior since the degree of distinguishability varies with $\lambda$. The transition point being roughly at $\lambda=0.5$.

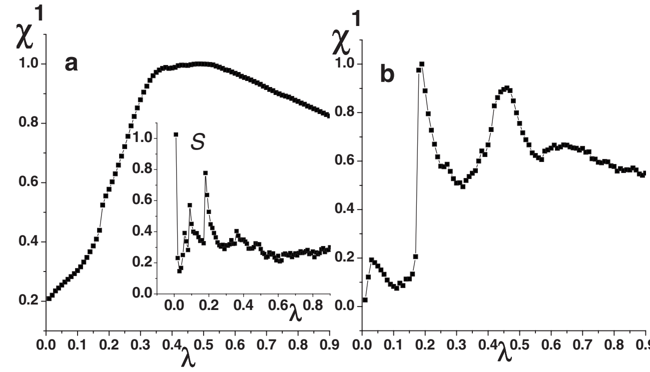


FIG. 1. Plots of (normalized) $\chi_\rho^{(1)}(T,\lambda)$ corresponding to $T=1$ (a) and $T=10$ (b). The state is $\rho=\mathbb{1}/d$. Inset: plot of the relative entropy $\mathcal{S}(P_\lambda \| P_W)$.

We now analyze the numerical results for $\chi_\rho^{(1)}(t,\lambda)=\Sigma_{n\neq m}\rho_{n,n}|\langle n|H'|n\rangle|^2\delta_t(\Delta_{n,m})$, where $H'=\partial_\lambda H=(a^\dagger+a)(J_+ + J_-)/\sqrt{2j}$ implements the perturbation to $H$. We have first chosen $\rho=\mathbb{1}/d$, i.e., scalar product in Eq. (1) is Hilbert-Schmidt, this corresponds to an infinite-temperature thermal state over the truncated working space. In Figs. 1 and 2 we show $\chi_\rho^{(1)}(t=T,\lambda)$ [normalized to its maximal value $\max_\lambda \chi_\rho^{(1)}(T,\lambda)$] for different $T$s. These plots clearly show that for growing values of $T$ the behavior of $\chi_\rho^{(1)}(t,\lambda)$ changes significantly. For $T>1$ the response of the system to the infinitesimal change in parameter $\lambda$, as described by $\chi_\rho^{(1)}(t,\lambda)$, clearly exhibits two markedly different behavior for $\lambda<0.5$ and $\lambda>0.5$. Indeed this behavior is coherent with the conjecture described in the previous paragraphs: the long time response of the system is characterized by the sensitivity of $\chi_\rho^{(1)}(t,\lambda)$ to the $S\to 0$ part of the spectrum. In particular, in the regular region, consistently with the relative entropy analysis, the behavior is nonuniform with $\lambda$ and the (nonuniform) presence of level crossings give rise to a pronounced response of $\chi_\rho^{(1)}$ characterized by a sequence of high peaks. In contrast, in the chaotic region the level repulsion phenomenon is reflected in a response which is characterized

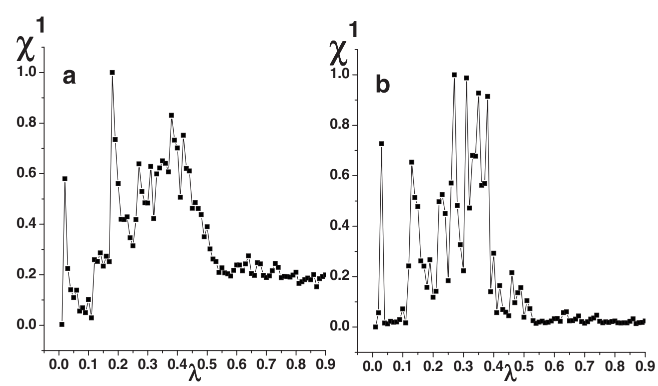


FIG. 2. Plots of the (normalized) $\chi_\rho^{(1)}(T,\lambda)$, (a) $T=100$ and (b) $T=1000$. The state is $\rho=\mathbb{1}/d$. For $\lambda<0.5$, $\chi_\rho^{(1)}(T,\lambda)$, due to level crossings, is characterized by a sequence of peaks. For $\lambda>0.5$ the peaks disappear (level repulsion) and $\chi_\rho^{(1)}(T,\lambda)$ describe the behavior of a system resilient to perturbations.

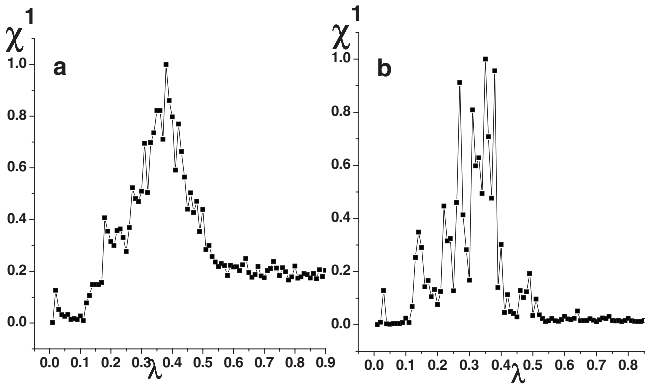


FIG. 3. Plots of (normalized) $\chi_\rho^{(1)}(t,\lambda)$; (a) $t=100$ and (b) $t=1000$. The state is $\rho=\exp(-\beta H)/\mathrm{Tr}[\exp(-\beta H)](\beta=0.014)$. See text and Fig. 2 for comments.

by the absence of a sharp peaks. Indeed, for $\lambda>0.5$, the response is low and constant thus reflecting the resilience of the chaotic system to the external perturbations.

We finally complete our analysis by showing in Fig. 3 how $\chi_\rho^{(1)}(t,\lambda)$ behaves by choosing in Eq. (1) the thermal state $\rho=\exp(-\beta H)/\mathrm{Tr}[\exp(-\beta H)]$ with finite $\beta$. In principle the introduction of a finite temperature by inducing a relative weight among the energy levels could spoil their chaotic vs regular distributions. Here we have chosen the inverse temperature $\beta=0.014$; this corresponds to fixing the ratio between the thermal probabilities relative to the ground and the highest energy state $\exp[-\beta(E_{min}-E_{max})]\approx 0.05$. Moreover we checked that for this $\beta$ the spectral features of $P(S)$ relevant to chaos are not washed out. Figure 3 shows that the $T=100$ and $T=1000$ behavior of $\chi_\rho^{(1)}(t,\lambda)$ is again markedly different in the two regions; the response of the system is enhanced by the presence of level crossings in the regular region, while it is suppressed in the chaotic region. These results are again consistent with our conjecture and suggest that the operator fidelity metric might well work as indicator for quantum chaos in appropriately chosen temperature intervals.

## V. CONCLUSIONS

In this Brief Report we have tackled the problem of characterizing the chaotic properties of quantum systems by means of an information-geometrical tool: the operator fidelity metric. The results of our analysis are twofold. At a purely analytical level, by means of techniques drawn by random matrix theory, we have formulated and substantiated the conjecture that the operator fidelity metric may provide a powerful indicator for the smooth regular to chaotic crossover. We have thus confirmed from an information-geometrical point of view the idea that a generic chaotic system, at variance with a regular system, is resilient with respect to random perturbations. We have then shown how the operator fidelity approach can be used as a tool to numerically identify the regular to chaotic crossover in a relevant many-body system, i.e., the Dicke model. We believe that the results obtained in this Brief Report, while preliminary, are promising and give rise to several interesting questions. Assessing the generality and efficiency of the methods we introduced and unveiling their relations with other approaches to quantum chaos, e.g., [11] is the subject of ongoing investigation.


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[20] To prove the result, express the matrix elements of both sides in terms of the eigenoperator basis $|\hat{\Psi}_{n,m}\rangle$ and perform the Gaussian averages.